Effect of Interaction on a Bosonic Condensate Confined in a Harmonic Trap

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Study of Bose-Einstein condensation (BEC) is undergoing world-wide explosion both experimentally and theoretically. It is of particular interest to the theorists to investigate a weakly interacting condensed Bose gas, externally confined by various geometries. In this work we report the role of inter-atomic interaction on the condensed phase of a Bose gas, confined in a harmonic trap.

1. Introduction

Bose-Einstein Condensation (BEC) is purely a quantum-statistical phase transition - a remarkable property by virtue of which, Bosons condense to the same quantum state below a certain transition temperature. This phase transition is unique in that it occurs even in the absence of interactions. It was first predicted by Einstein for a system of non-interacting Bosons. On the other hand, in real life particles will always interact and even the weakly interacting Bose gas behaves differently from the ideal Bose gas, qualitatively [1]. It is now well known that the interaction plays a crucial role in forming a stable many-body state at low temperatures. An important feature of an interacting Bose gas is that it presents a spectral gap from the ground state to the low-lying quasi-particle excitation energy spectrum. The existence of the spectral gap fixes the condition for BEC that the inter-atomic interactions should be repulsive definite.

In the ground state of the interacting gas, not all particles are in the lowest momentum state because the two-body interaction mixes into the ground state components with atoms in other states. In liquid ^{4}He, a system of strongly interacting Bosons, the density of the condensate, even at very low temperature and pressure, is reduced to approximately 10% of the total number density . Studies [2] have been made for several years to determine the depletion of the ground state. Pethick and Smith [2] have used the Bogoliubov approximation to determine the depletion of the condensate and the change in ground-state energy of an interacting homogeneous Bose gas at zero temperature. Fetter and Walecka [2], have calculated the depletion of the ground state of an interacting Bose gas at zero temperature using field theoretical methods within the framework of perturbation theory.

In the presence of confinement, interactions affect the character of BEC owing to the inhomogeneity introduced in the system, which facilitates Bose condensation. In most experiments that have been carried out in harmonic traps, the ground state depletion of the condensed atoms is of the order of one percent. Recent experiments on ^{85}Rb near a Feshbach resonance achieved very large values of the scattering length corresponding to a depletion of 10%. In our present study, we discuss how temperature and interaction affect the ground state density of a confined Bose gas.

2. Ideal Bose gas confined by an external isotropic harmonic potential

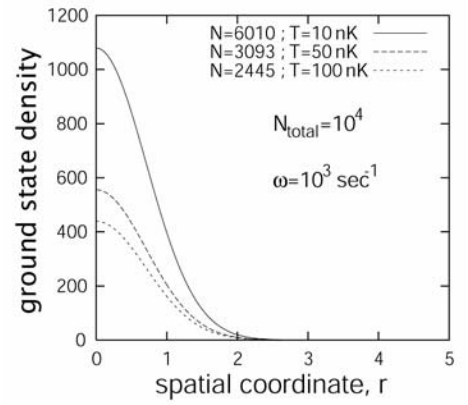
Ideal Bose gas confined by a harmonic potential has been studied [3] by several authors in the thermodynamic limit as well as in finite systems. As an illustration, we consider a finite system of 10^4 non-interacting Bosons, confined by a 3D isotropic harmonic potential given

by $V(r) = m\omega^2 r^2 /2$. Here m is the mass of each Boson and ω is the oscillator frequency. The discrete energy levels of the harmonic potential are given by : $E_{n1+n2+n3} = \hbar\omega(n_1+n_2+n_3+3/2)$, where $n_1+n_2+n_3 = 0,1,2\ldots$ and $E_0= (3/2)\hbar\omega$.
In the grand canonical ensemble at temperature T, the total number of particles N is given by:

$$N = \sum_{m_x, m_y, m_z=0}^{\infty} \frac{1}{e^{[\beta E'_{n_1,n_2,n_3}+\beta(E_0-\mu)]} - 1}$$

where $E'_{n1+n2+n3} = E_{n1+n2+n3} - E_0$, $\beta = 1/kT$ and μ is the chemical potential which is determined from the condition that the total number of particles in the system is conserved. As a result of the trapping potential the density becomes inhomogeneous and Bose-Einstein condensation first sets in, at the centre of the trap where the density is maximum. When BEC takes place the quantum state that can be occupied by the condensate is the oscillator ground state. At sufficiently low temperatures, the ground state is macroscopically occupied and the size of the condensate is fixed by the harmonic oscillator length $a_{ho}= (\hbar/m\omega_{ho})^{1/2}$.

At a finite temperature of 10 nK, the particles from the ground state are excited thermally to higher energy levels and the ground state number is reduced consequently. As the temperature is increased further to 50 nK and 100 nK, the ground state number gets further reduced. Thus we see that, increasing the temperature results in the depletion of more number of particles from the ground state. This is shown in the figure below.

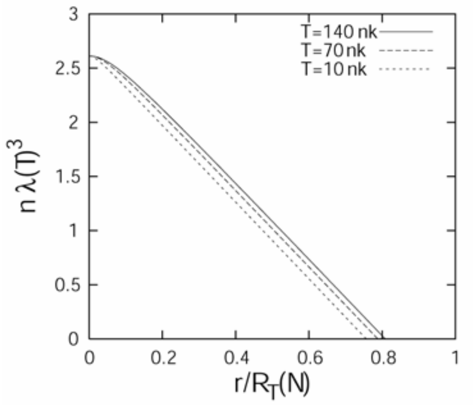


3. Interacting Bose gas confined by an external isotropic harmonic potential

BEC considered in the above section is unrealistic and therefore we consider an interacting Bose gas in the weak coupling limit: $a/a_{ho} << 1$, where 'a' is the s-wave scattering length. The gas is dilute and hence $na^3 << 1$. Here we discuss the finite temperature effects on the degeneracy parameter $n\lambda^3$ keeping the strength of the interaction fixed. For this we begin with the density profile of the gas in the quantum regime, which in the mean field approximation is given by [4] :

$$n(\mathbf{r}) = n_c^0 + \frac{8\pi a}{\lambda^4} - \sqrt{\left[n_c^0 - n(0) + \frac{8\pi a}{\lambda^4}\right]^2 + \frac{4\pi}{\lambda^6}\beta V(\mathbf{r})}$$

where $\beta V(\mathbf{r}) = (r/R_T)^2$. By plotting $n\lambda^3$ versus r/R_T, where R_T is the thermal range, we see that as the temperature is reduced from 140 nK to 10 nK the degeneracy parameter gets reduced. This implies that the number of particles in the ground state has been reduced due to thermal effects. This is shown in the figure below.

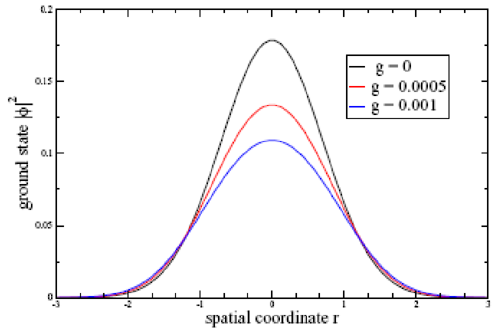


The simplest approximation which one studies when interactions are present in a confined Bose system is the mean field approximation by Bogoliubov. The function $\phi(\mathbf{r},t)$ is a 'classical' field having the meaning of an order parameter and is often called the "macroscopic wave function of the condensate". The equation for the order parameter is the famous GP equation, which is :

$$i\hbar\frac{\partial\phi(\mathbf{r},t)}{\partial t} = \left(\frac{-\hbar^2\nabla^2}{2m} + V_{trap}(\mathbf{r}) + g|\phi(\mathbf{r},t)|^2\right)\ \phi(\mathbf{r},t)$$

where 'g' is the coupling constant, $4\pi\hbar^2 a/m$.

Numerical solutions of the GP equation [5] for the ground state wave function of a harmonically trapped weakly interacting condensate with repulsive interactions have been obtained by a variety of groups for both the isotropic and the anisotropic cases. Here, in the figure below we show the role of the inter-atomic interaction in the depletion of the condensate at zero temperature, in the presence of an isotropic harmonic trap. Compared to the bare harmonic oscillator ground state wave function, which has the form of a Gaussian, the wave function of the condensate in the presence of repulsive interactions deviates from the Gaussian, as a consequence of the condensate being depleted and spatially broadened.



4. Results

We have shown that at finite temperatures the number of particles in the ground state is reduced due to increase in temperature, for both the non-interacting as well as the interacting systems and that the density in the central region of the trap, at zero temperature, is reduced due to the presence of interactions. We have shown the depletion at finite temperature, quantum mechanically for the non-interacting confined system and perturbatively for the interacting confined system in the weak coupling limit.

References

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